Chapter NLP:I

I. Natural Language Processing Basics

- ❑ String Processing
- ❑ Grammars
- ❑ Regular Expressions

String Processing

Overview

String processing forms the basis of many tasks, algorithms, and evaluation methods in NLP.

String problems:

- Sorting
- Manipulation
- Exact / inexact matching
  Search and indexing, similarity and distance
- Alignment
  Longest common subsequence
- Parsing
  Splitting
- Compression

Data structures:

- Buffer
- Inverted index
- Trie, suffix tree, suffix array

Computational models:

- Finite-state machines
- Dynamic programming

Remarks:

- String processing combines methods from formal language theory, compilers, and bioinformatics.

Grammars

Terminology

- Alphabet Σ.

  An alphabet Σ is a finite non-empty set of letters or symbols.

- Word w.

  A word w is a finite sequence of symbols from Σ. The length of a word $|w|$ is the number of its symbols.

  ε denotes the empty word; it is the only word with length 0.

  Σ^* denotes the set of all words over Σ.

- Language L.

  A language L is a set of words over an alphabet Σ.

- Grammar G.

  A grammar G is a calculus for defining a language – that is, a set of rules that can be used to derive words. The language belonging to G consists of all derivable, terminal words.

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Remarks:

- When defining language characteristics, a distinction is made between different levels of abstraction. Level 1 deals with the notation of symbols, while Level 2 deals with the syntactic structure of the language.
  [Exkurs: Programmiersprachen]
- To distinguish between the different levels of grammar usage, the following terms can be used:
  - Level 1: Alphabet, symbol, word, language
  - Level 2: Vocabulary, token, sentence, language
- The words {alphabet, vocabulary}, {symbol, token} or {word, sentence} are the respective equivalents of the elementary symbolic level and the syntactic level.

Grammars

Definition 1 (Grammar)

A grammar is a quadruple $G = (N, \Sigma, P, S)$ with

N = finite set of non-terminal symbols

Σ = finite set of terminal symbols, $N \cap \Sigma = \emptyset$

P = finite set of productions or rules

$$P \subset (N \cup \Sigma)^* \, N \, (N \cup \Sigma)^* \times (N \cup \Sigma)^*$$

S = start symbol, $S \in N$

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S = start symbol, $S \in N$

Remarks:

- A rule consists of a left side (premise) and a right side (conclusion), each of which is a word consisting of terminals and non-terminals. The left side must contain at least one non-terminal, and unlike the left side, the right side can also be the empty word. [Wikipedia]
- A rule can be applied to a word consisting of terminals and non-terminals, whereby any occurrence of the left side of the rule in the word is replaced by the right side of the rule: $w \to w'$.
- Given the rule $w \to w'$, then w, w' are in the so-called *transitive relation* $\to_G$. A sequence of rule applications is called a *derivation*.

Grammars

Definition 2 (Generated Language)

The language $L(G)$ generated by a grammar $G = (N, \Sigma, P, S)$ contains exactly those words that consist only of terminal symbols and can be derived from the start symbol with a finite number of steps:

$$L(G) := \{w \in \Sigma^* \mid S \to_G^* w\}$$

$\to_G^*$ denotes the arbitrary application of the productions in G, i.e., the reflexive-transitive hull of the transition relation $\to_G$.

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Example:

$G = (N, \Sigma, P, S)$ mit $N = \{S, A, B\}$, $\Sigma = \{a, b\}$ und folgenden Produktionen:

$$\begin{array}{lcl} S & \rightarrow & ABS \\ S & \rightarrow & \varepsilon \end{array} \qquad \begin{array}{lcl} BA & \rightarrow & AB \\ BS & \rightarrow & b \\ Bb & \rightarrow & bb \end{array} \qquad \begin{array}{lcl} Ab & \rightarrow & ab \\ Aa & \rightarrow & aa \end{array}$$

Remarks:

- It is conventional to use uppercase letters to denote non-terminal symbols and lowercase letters to denote terminal symbols.
- Another grammar that generates the same language as in the example is:
  $N = \{S, A, B\}$, $\Sigma = \{a, b\}$, $P = \{S \to aSb, S \to \varepsilon\}$

Grammars

Chomsky Hiearchy

Grammars are divided into four classes based on the complexity of the languages they generate.

- Type 0 $\sim$ recursively enumerable.
- Type 1 $\sim$ context-sensitive.
- Type 2 $\sim$ context-free.
- Type 3 $\sim$ regular.

Grammars

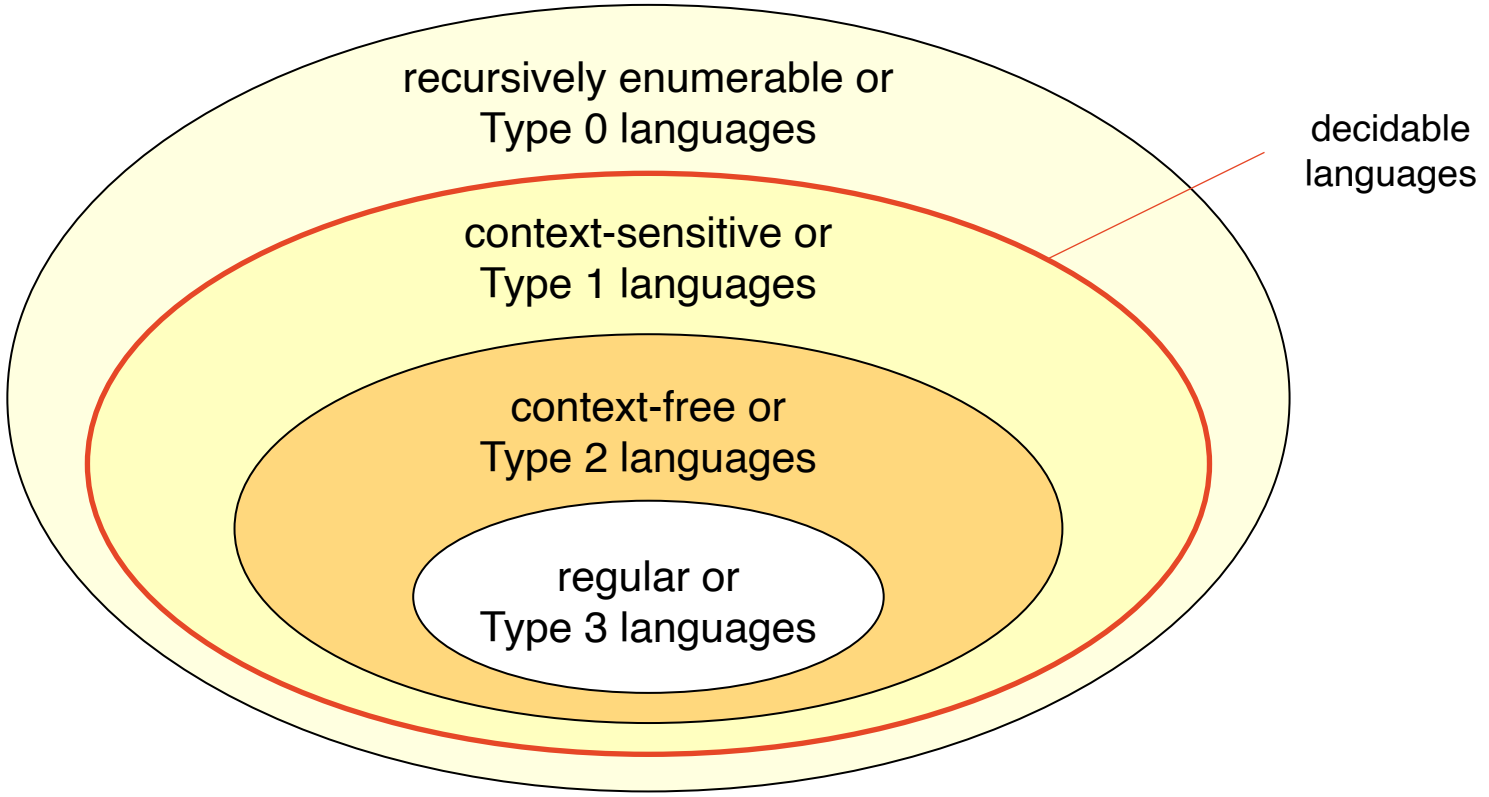
Chomsky Hiearchy

Grammars are divided into four classes based on the complexity of the languages they generate.

- Type 0 $\sim$ recursively enumerable.
  There are no restrictions for the rules in P.
- Type 1 $\sim$ context-sensitive.
  For all rules $w \to w' \in P$ holds: $|w| \leq |w'|$
- Type 2 $\sim$ context-free.
  For all rules $w \to w' \in P$ holds: w is a single variable; i.e., $w \in N$.
- Type 3 $\sim$ regular.
  The grammar is of Type 2, and for all rules, $w \to w'$ additionally holds: $w' \in (\Sigma \cup \Sigma N)$, i.e., the right-hand sides of the rules consist either of a terminal symbol or of a terminal symbol followed by a non-terminal.

Grammars

Chomsky Hierarchy



Definition 3 (Language of Type)

A language $L \subseteq \Sigma^*$ is called a Type 0 (Type 1, Type 2, Type 3) language if there exists a Type 0 (Type 1, Type 2, Type 3) grammar G such that $L(G) = L$.

Remarks:

- The Chomsky hierarchy represents a hierarchy with genuine subset relationships:
  Type 3 $\subset$ Type 2 $\subset$ Type 1 $\subset$ Type 0
- All languages of types 1, 2, or 3 are decidable:
  - $\rightarrow$ The decision problem if a word belongs to a language is decidable for all languages of Type 1, 2, or 3.
  - $\rightarrow$ There is an algorithm that, given a grammar G and a word w, decides in finite time if $w \in L(G)$ or not.
- The set of Type 0 languages is identical to the set of recursively enumerable or semi-decidable languages. Therefore, there are Type 0 languages that are not decidable.

  There are countably infinite grammars for generating a recursively enumerable language (Type 0)
- In compiler theory and natural language processing, Type 3 languages and grammars (lexical analysis, tokenization) and Type 2 languages and grammars (syntactic structure analysis) play a central role.

Grammars

Calculi for Regular Languages

Different calculi for generating words in a regular language:

(a) finite acceptor or automaton

(b) regular expressions

(c) Type 3 grammar

(d) Specifying finitely many equivalence classes (of the Nerode relation)

Grammars

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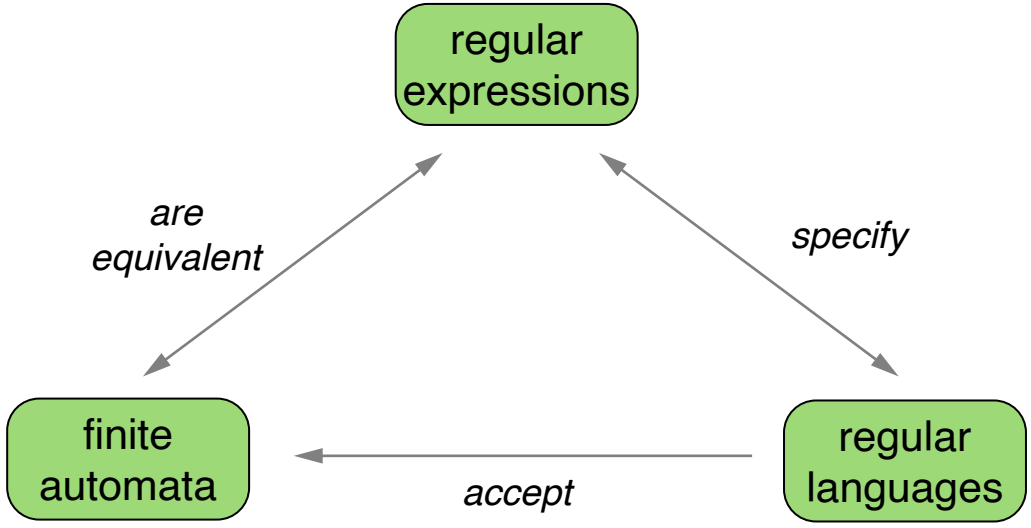

[Haenelt 2005, Jurafski/Martin 2000]

Grammars

Summary

	Language generation calculi
Type 0	Type 0 grammar Turing machine
Type 1	context-senstive grammar linear bounded Turing machine [Wikipedia]
Type 2	context-free grammar pushdown automaton
Type 3	regular grammar (Type 3 grammar) deterministic/non-deterministic finite automaton regular expression

Grammars

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	Complexity of the word problem [Wikipedia]
Type 0	undecidable
Type 1	exponential complexity, NP-hard
Type 2	$O(n^3)$
Type 3	linear complexity

Chapter NLP:I

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- Grammars
- Regular Expressions

Regular Expressions [Kastens]

Syntax

A regular expression R can be composed recursively as follows. F and G denote regular expressions.

	R	**Semantics**
1.	a	The letter a
2.	FG	Concatenation
3.	$F \mid G$	Alternation
4.	(F)	Grouping
5.	F^+	Non-empty sequence of words from $L(F)$
6.	F^*	Arbitrary sequence of words from $L(F)$
7.	F^n	Sequence of n words from $L(F)$
8.	ε	The empty word

Regular Expressions [Kastens]

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A regular expression R can be composed recursively as follows. F and G denote regular expressions.

	R	Language $L(R)$	Semantics
1.	a	$\{a\}$	The letter a
2.	FG	$\{fg \mid f \in L(F),\ g \in L(G)\}$	Concatenation
3.	$F \mid G$	$\{f \mid f \in L(F)\} \ \cup \ \{g \mid g \in L(G)\}$	Alternation
4.	(F)	$(L(F))$	Grouping
5.	F^+	$\{f_1 f_2 \ldots f_n \mid f_i \in L(F),\ n \geq 1,\ i = 1, \ldots, n\}$	Non-empty sequence of words from $L(F)$
6.	F^*	$\{\varepsilon\} \ \cup \ L(F^+)$	Arbitrary sequence of words from $L(F)$
7.	F^n	$\{f_1 f_2 \ldots f_n \mid f_i \in L(F),\ i = 1, \ldots, n\}$	Sequence of n words from $L(F)$
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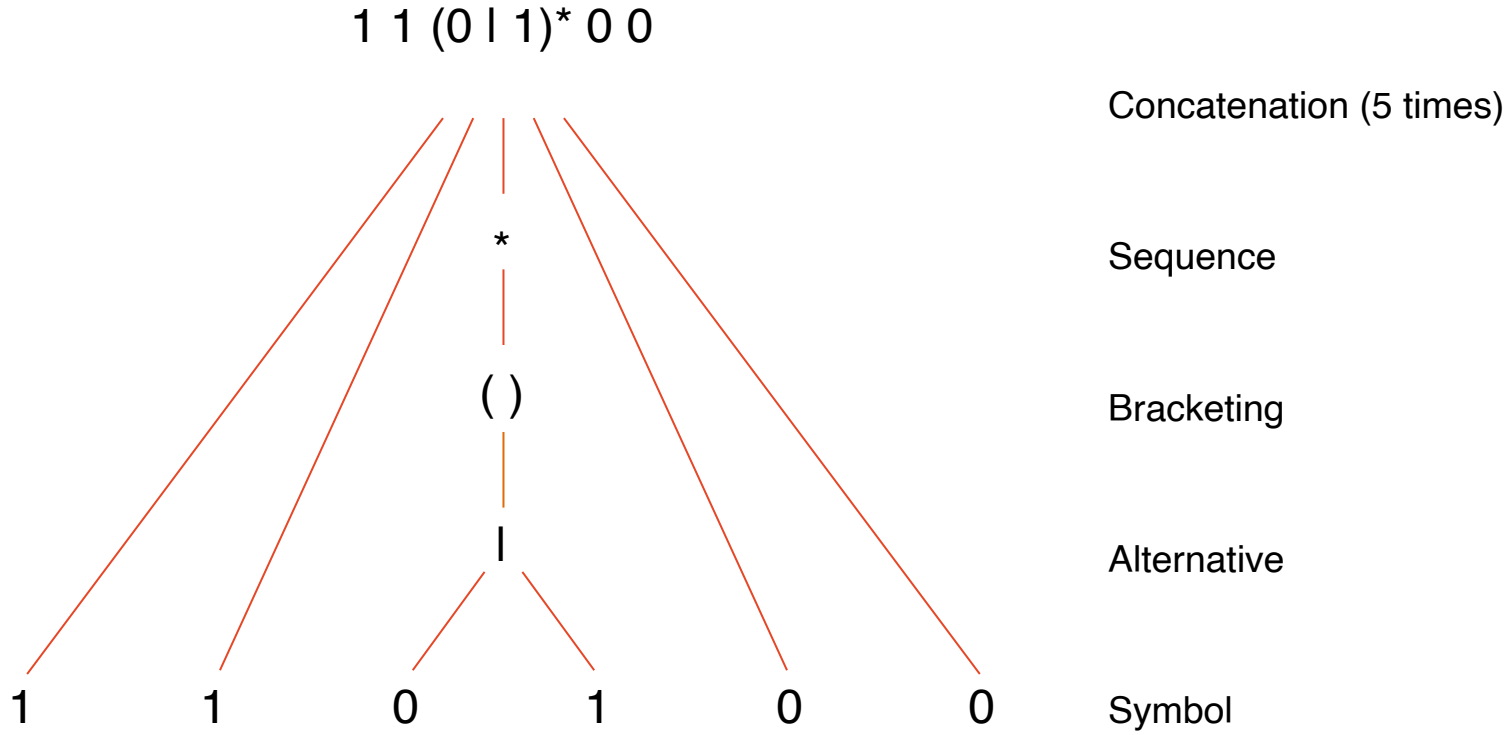
Regular Expressions [Kastens]

Examples



Every word in the language of this regular expression consists of two ones, followed by any number of zeros or ones, followed by two zeros.

Regular Expressions [Kastens]

Examples (continued)

R	**Name of language** $L(R)$	**Words from** $L(R)$
(a | b) (c | d | ε)	*Abc*	ac, bc, ad, bd, a, b
Dear(ε | est) (Sir | Madam)	*Salutation*	Dear Sir

Regular Expressions [Kastens]

Examples (continued)

R	**Name of language** $L(R)$	**Words from** $L(R)$
(a | b) (c | d | ε)	*Abc*	ac, bc, ad, bd, a, b
Dear(ε | est) (Sir | Madam)	*Salutation*	Dear Sir
0 | 1 | . . . | 9	*Digit*	7
a | b | . . . | z	*sLetter*	x
A | B | . . . | Z	*cLetter*	B
sLetter | *cLetter*	*Letter*	m, N

Regular Expressions [Kastens]

Examples (continued)

R	**Name of language** $L(R)$	**Words from** $L(R)$
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a | b | . . . | z	*sLetter*	x
A | B | . . . | Z	*cLetter*	B
sLetter | *cLetter*	*Letter*	m, N
Letter (*Letter* | *Digit*)*	*Label*	Maximum, min7, a
Digit $^{+}$.*Digit* ^{2}	*MoneyAmount*	23.95, 0.50
(*cLetter* | *cLetter* ^{2} | *cLetter* ^{3})– (*cLetter* | *cLetter* ^{2})– (*Digit* | *Digit* ^{2} | *Digit* ^{3} | *Digit* ^{4})	*LicensePlatesDE*	PB–AS–0815
1^{3} (1 | 0)* 0^{3}	*Dual*	1111000, 1111101010000

Regular Expressions

Examples (continued)

An important use of regular expressions in languages used for natural language processing is the specification of text patterns.

Example: Display of all file names of the form

$$\text{“winter-term(0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9)}^2\text{.html”}$$

- Unix-Shell.

```
ls winter-term[0-9][0-9].html
```

- PHP.

```
$d = "[0-9]";
preg_match("/winter-term$d$d\.html/", $files)
```

Remarks:

- If names of regular expressions are used in other regular expressions, they must be identified as part of the meta language.

  Here: Use of italics.

- Each scripting language for text processing uses a different syntax to specify regular expressions; the construction principles and power are comparable.
- The specification of regular expressions in PHP is taken from the Perl scripting language.

Regular Expressions

Definition 4 (Exact Matching Problem [Gusfield 1997])

Given a string R called the pattern and a longer string T called the text, the exact matching problem is to find all occurrences, if any, of pattern R in text T.

Example:

- Given $R = aba$ and $T = bbabaxababay$, then R occurs in T starting at locations 3, 7, and 9.
- Two occurrences of R overlap.

Importance of the exact matching problem:

- Exact matching is solved for searching local files and data.
- Main bottleneck is the speed with which the data can be read into memory.
- Speed rests on “preprocessing” pattern R (or text T) to enable skipping during search.

Regular Expressions

Perl Compatible Regular Expressions (PCRE) [pcre.org, PHP, Python, JavaScript, Java, .NET]

Many programming languages use a Perl-like syntax for regular expressions:

"*Delimiter Regular_Expression Delimiter* [*Flags*]"

Pattern

- The delimiter must be a non-alphanumeric character.
- Optional flags influence the matching strategy. [pcre.org, PHP, Python, JavaScript, Java, .NET]
- Search and replace functions with regular expressions. [pcre.org, PHP, Python, JavaScript, Java, .NET]

Regular Expressions

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- Search and replace functions with regular expressions. [pcre.org, PHP, Python, JavaScript, Java, .NET]

Examples: (PHP)

```
                    Pattern          Text
echo preg_match("/def/" , "defabcdef" );    ⇝ 1

echo preg_match("=def=", "defabcdef");      ⇝ 1
```

Regular Expressions

Perl Compatible Regular Expressions (continued) [Regular Expressions (Syntax)]

A regular expression R can be composed recursively as follows. Let F and G denote regular expressions and $L(F)$, $L(G)$ their languages over the alphabet Σ.

	R [pcre.org]	**Semantics**
1.	a	The letter a
	.	Any symbol from Σ (except line break)
	$[\Sigma']$	Character class: Any symbol from $\Sigma' \subseteq \Sigma$
	$[\hat{}\Sigma']$	No symbol from character class $\Sigma' \subseteq \Sigma$
	$\backslash\Sigma'$	Escaped letter from a controlled $\Sigma' \subset \Sigma$, denoting a character class
2.	FG	Concatenation
3.	$F \mid G$	Alternation
4.	(F)	Grouping or capturing
	$F?$	F is optional (same semantics as $F \mid \varepsilon$)
5.	$F+$	Non-empty sequence of words from $L(F)$
6.	$F*$	Arbitrary sequence of words from $L(F)$
7.	$F\{n\}$	Sequence of n words from $L(F)$
	$F\{m,n\}$	Sequence with at least m and at most n words from $L(F)$
	$\hat{}$ or \$	Assertion: Beginning or end of the text string (nothing may precede or follow it)

Regular Expressions

PCRE: Character Classes [pcre.org]

Character classes match a single token from a list of characters.

R	**Semantics**	**Example**
$[...]$	Positive character class matches listed character	`[aeiou]` matches vowels
$[\hat{}...]$	Negative character class matches only unlisted characters	`[^aeiou]` matches anything except for vowels
$[x-y]$	Range: matches any letter between x and y inclusive	`[a-zA-Z]` matches alphabetic characters
`\d`	Matches any decimal digits. Short for `[0-9]`	
`\D`	Matches all except decimal digits. Short for `[^0-9]`	
`\s`	Matches white space characters. Short for `[\t\n\r\f\v ]`	
`\S`	Matches all except white spaces. Short for `[^\t\n\r\f\v ]`	
`\w`	Matches alphanumeric characters. Short for `[a-zA-Z0-9]`	
`\W`	Matches non-alphanumeric characters. Short for `[^a-zA-Z0-9]`	

Remarks:

- `]` needs to be escaped inside character classes to be matched: `[^]]` does not match `a` but, e.g., `]]`.
- The range in a character class denotes a series of codepoints.
  As such, `[--2]` is valid and identical to `[-./012]`. `[A-z]` matches not only alphabetic characters.
- Character classes cannot be nested but `\d`, `\D`, `\s`, ... can be used inside character classes:
  - `[^[a-z][A-Z]]` matches the regex `[^[a-z]`, `[A-Z]`, and `]` in sequence.
  - `[^\s\w]` matches any character that is neither a space, nor a word-character.

Regular Expressions

PCRE: Groups and Backreferencing [pcre.org]

Groups give precedence to subformulas and can store (capture) matches for later reference.

R	**Semantics**	**Example**
`(`F`)`	Capture Group: Groups F and stores the matched value	`(ab)+` matches `ab`, `abab`, ...
`(?:`F`)`	Non-capturing group: Groups F	`(?:ab)+` matches `ab`, `abab`, ...
`(?<`*name*`>`F`)`	Named capture group: Groups F and stores it under a *name*	
`\`n	Backreference the n-th capture group	`(\w+)\1` matches fully reduplicated words
`\k{`*name*`}`	Backreference capture group named "*name*"	`(?<word>\w+)\k{word}` is the same as above

- Capture groups store the last match.
  The regex `([ab])+\1` matches `abb` but not `aba`.
- Named capture groups can extract information from different positions in the regex.
  The regex `\d\d.(?<month>\d\d).\d\d|(?<month>\d\d)/\d\d/\d\d` extracts the month from `16.01.70` and `01/16/70`.

Remarks:

- The `\n` syntax is highly ambiguous:
  - If n is an octal number (`\1`, `\2`, ..., `\7`, `\10`, ...), it matches the character point with value n.
  - If n is a single non-octal digit (`\8`, `\9`), it matches the respective character.
  - If n is a valid index for a capture group, the above is irrelevant and the capture group is backreferenced.

  `\8` matches the character `8` but `()()()()()()()()\8` only matches the empty word.
  `\49` gives an error because there is no 49th capture group but `\50` matches `(`.

➜ PCRE introduces `\gn` or `\g{n}` to reference capture groups and `\on` for octal codepoints. These are less common in other flavors of regex.

- Regular expressions compatible with PCRE2 are more powerful than regular grammars.
  The language $L = \{ww \mid w \in \{0,1\}^*\}$ is not context free but can be decided by the regex `([01])*\1`.

Regular Expressions

PCRE: Quantifiers [pcre.org]

Quantifiers allow for repeated matching of a regular expression F.

R	**Semantics**	**Example**
F{m}	Sequence of m words from $L(F)$	`\d{2}` matches exactly two digits
F{m,}	Sequence of at least m words from $L(F)$	`\d{2,}` matches two or more digits
F{m,n}	Sequence of at least m and at most n words from $L(F)$	`\d{2,5}` matches two to five digits
F{,n}	Sequence of at most n words from $L(F)$ *(since PCRE2 10.43)*	`\d{,10}` matches up to ten digits
F?	Equivalent to $F\{0,1\}$	`\w?` matches at most one word-character
F*	Equivalent to $F\{0,\}$	`\w*` matches any number of word-characters
F+	Equivalent to $F\{1,\}$	`\w+` matches one or more word-characters

- Default. Greedy matching: as much as possible and backtracking until a match is found.
  `\d*\d\d` tries to match `1234`

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- Default. Greedy matching: as much as possible and backtracking until a match is found.
  `\d*\d\d` tries to match `1234` and but `\d\d` matches as well → success!

Regular Expressions

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- Modify the matching behavior by adding `?` (lazy) or `+` (possessive) behind the quantifier.
  `\d+?` matches as few digits as possible, but at least one. `\d++` matches greedily without backtracking

Strings	R			
	`".*"`	`".*?"`	`".*+"`	`"[^"]*+"`
`"hi"`	{`"hi"`}	{`"hi"`}	∅	{`"hi"`}
`"foo" "bar"`	{`"foo" "bar"`}	{`"foo"`,`"bar"`}	∅	{`"foo"`,`"bar"`}

Remarks:

- Possessive matching is more efficient than greedy or lazy matching because no backtracking is performed.
- Possessive matching is a special case of an atomic group (written `(?>`F`)`). Atomic groups cannot be backtracked.
  The regex `(?>a|aa)a` matches `aa` but not `aaa` because for the latter it would initially match `a` for the capture group (the first matching option in the alternative) and then have to backtrack when the end of the string is reached.
  What strings would `(?>aa|a)a` match?
- F`?+`, F`*+`, and F`++` are short for `(?>`F`?)+`, `(?>`F`*)+`, and `(?>`F`+)+` respectively.

Regular Expressions

PCRE: Assertions [pcre.org]

Assertions test if a regex would match without matching it.

R	**Semantics**
`(?=`F`)`	(Positive) lookahead: Asserts that F matches at the current location.
`(?!`F`)`	Negative lookahead: Asserts that F does not match at the current location.
`(?<=`F`)`	(Positive) lookbehind: Asserts that F matches right before the current location.
`(?<!`F`)`	Negative lookbehind: Asserts that F does not match right before the current location.
	`(?<=\s)(?<!$)\d+` matches any number in the text that is not preceded by a $.
`^`	Begin of string anchor: Asserts the start of the input sequence.
`$`	End of string anchor: Asserts the end of the input sequence.
	`^[01]+\$` checks that the input entirely represents a binary number
`\b`	Asserts a word boundary. Short for `(?<=\W)(?=\w)|(?<=\w)(?=\W)`
`\B`	Asserts the opposite of `\b`. Short for `(?<=\W)(?=\W)|(?<=\w)(?=\w)`
	`\b[Tt]he\b` matches the word "the" but not if it occurs inside another word as in "atheist".
	`\Bing\b` matches the suffix -ing. For example in "climbing" or "sing" but not in "singer".

- `^`, `$`, `\b`, `\B` are simple assertions and `(?=`F`)`, `(?!`F`)`, `(?<=`F`)`, `(?<!`F`)` are assertion groups.
- Simple assertions cannot be followed by quantifiers.
  For example, `\b+` is illegal but `(\b)+` is allowed.